Visualization of topological structure and chiral condensate [*]

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We perform a mutual analysis of the topological and chiral vacuum structure of four-dimensional QCD on the lattice at finite temperature. We demonstrate that at the places where instantons are present, amplified production of quark condensate takes place. It turns out for full QCD that the clusters of nontrivial chiral condensate have a size of about 0.4 fm corresponding to the instanton sizes in the same configurations.

It is well known that the zero eigenvalues of the fermionic matrix are related to the global topological charge Q of a gauge field configuration via the Atiyah-Singer index theorem. It is believed that the instantons as carriers of the topological charge might play a crucial role in understanding the confinement mechanism of four-dimensional QCD, if one assumes that they form a so-called instanton liquid [1]. Recently, it was demonstrated that monopole currents appear preferably in the regions of non-vanishing topological charge density [2,3]. It has been conjectured that both instantons and monopoles are related to chiral symmetry breaking [1,4]. This idea is further supported by the following results of a direct investigation of the local correlations of the quark condensate and the topological charge density.

For the implementation of the topological charge on a Euclidian lattice we restrict ourselves to the so-called field theoretic definitions which approximate the topological charge density in the continuum, $q(x) = \frac{g^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma} \,\mathrm{Tr}\left(F_{\mu\nu}(x)F_{\rho\sigma}(x)\right)$. We used the plaquette and the hypercube prescription. To get rid of quantum fluctuations and renormalization constants, we employed the Cabbibo-Marinari cooling method. Mathematically and numerically the local chiral condensate $\bar{\psi}\psi(x)$ is a diagonal element of the inverse of the fermionic matrix of the QCD action. We compute correlation functions between two observables $\mathcal{O}_1(x)$ and $\mathcal{O}_2(y)$

$$g(y - x) = \langle\mathcal{O}_1(x)\mathcal{O}_2(y)\rangle - \langle\mathcal{O}_1\rangle\langle\mathcal{O}_2\rangle \qquad (1)$$
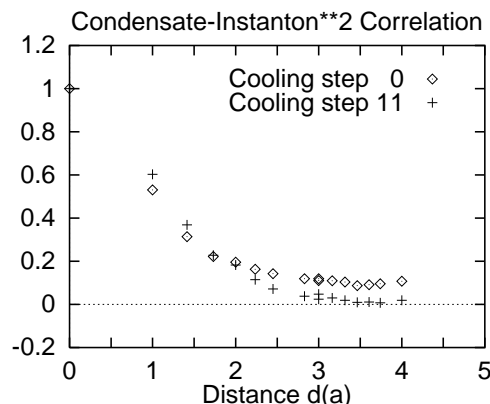
Figure 1. Correlation function of the quark-antiquark density and the topological charge density for 0 and 11 cooling steps. The correlations extend over two lattice spacings and indicate local coexistence of the quark condensate and topological objects.

and normalize them to the smallest lattice separation $d_{\min}$, $c(y-x) = g(y-x)/g(d_{\min})$. Since topological objects with opposite sign are equally distributed, we correlate the quark-antiquark density with the square of the topological charge density. Our simulations were performed for full $SU(3)$ QCD on an $8^3 \times 4$ lattice with periodic boundary conditions. Applying a standard Metropolis algorithm has the advantage that tunneling between sectors of different topological charges occurs at reasonable rates. Dynamical quarks in Kogut-Susskind discretization with 3 flavors of degenerate mass $m = 0.1$ were taken into account using the pseudofermionic method. We performed runs in the confinement phase at $\beta = 5.2$. Measurements were taken on 1000 con-
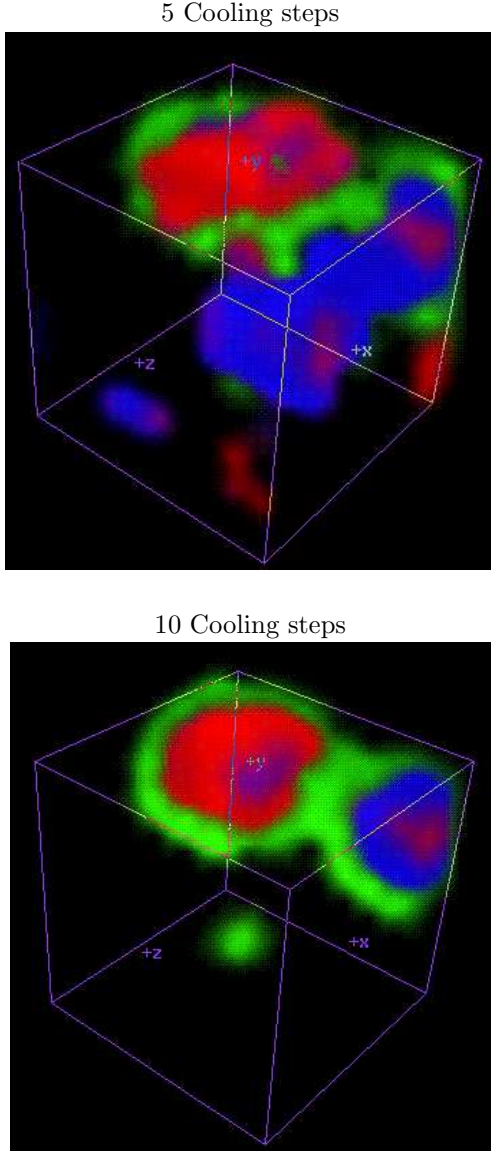
5 Cooling steps

10 Cooling steps

Figure 2. Cooling history for a time slice of a single gauge field configuration of $SU(3)$ theory with dynamical quarks. The dark and medium grey shades represent the positive and negative topological charge density respectively; the surrounding light grey tone the density of the quark condensate. It turns out that the quark condensate takes a non-vanishing value at the positions of the instantons.

figurations separated by 50 sweeps.

Figure 1 shows results for the correlation functions of Eq. (1) with $\mathcal{O}_1 = \bar{\psi}\psi(x)$ and $\mathcal{O}_2 = q^2(y)$. The $\bar{\psi}\psi q^2$-correlation function exhibits an extension of more than two lattice spacings, indicating nontrivial correlations. To gain information about the correlation lengths exponential fits to the tails of the correlation functions were performed. They show that an increasing number of cooling steps yields shorter correlation lengths. The corresponding screening masses are $\zeta = 0.59$ and $\zeta = 1.56$ in inverse lattice units for 0 and 11 cooling steps, respectively. They have to be interpreted as effective masses and reflect the effective gluon exchange and the vacuum structure of QCD.

It is assumed that the size ρ of a t'Hooft instanton $q_\rho(x) \sim \rho^4 (x^2 + \rho^2)^{-4}$ centered around the origin enters also into the associated distribution of the chiral condensate $\bar{\psi}\psi_\rho(x) \sim \rho^2 (x^2 + \rho^2)^{-3}$ [5]. To estimate ρ we fitted a convolution of the functional form $f(x) = \int \bar{\psi}\psi_\rho(t) q_\rho^2(x-t) \, dt$ to the data points. This was evaluated after 11 cooling steps where the configurations are reasonably dilute. Our fit yields $\rho(\bar{\psi}\psi q^2) = 1.8$ in lattice spacings. To check consistency we extracted from the qq-correlation a value of $\rho(qq) = 2.05$ which is in good agreement.

We now visualize densities of the quark condensate and topological quantities from individual gauge fields. The topological content of typical individual gauge field configurations was already studied for pure $SU(2)$ and $SU(3)$ theory. We found that at the local regions of clusters of topological charge density, which are identified with instantons, there are monopole trajectories looping around in almost all cases [2].

In Fig. 2 a time slice of a typical configuration from $SU(3)$ theory with dynamical quarks on the $8^3 \times 4$ lattice in the confinement phase is shown. We display the positive instanton density by dark grey shades and the negative density by a medium grey-tone if the absolute value $|q(x)| > 0.003$. The quark-antiquark density is indicated by a light grey shade whenever a threshold for $\bar{\psi}\psi(x) > 0.066$ is exceeded. By analyzing dozens of gluon and quark field configurations we found the following results. The topological
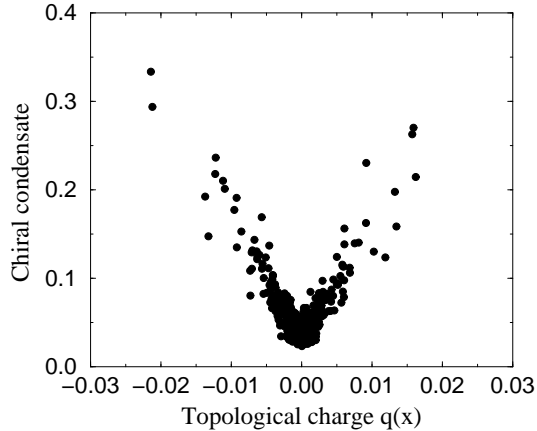
Figure 3. Scatter plot of $\bar{\psi}\psi(x)$ against $q(x)$ in the volume of the same gauge field configuration with 10 cooling steps as in Fig. 2. It reflects the separation into an instanton and antiinstanton.

charge is covered by quantum fluctuations and becomes visible by cooling of the gauge fields. For 0 cooling steps no structure can be seen in $q(x)$, $\bar{\psi}\psi(x)$ or the monopole currents, which does not mean the absence of correlations between them. After 5 cooling steps clusters of nonzero topological charge density and quark condensate are resolved. This particular configuration possesses a cluster with a positive and a negative topological charge corresponding to an instanton and an antiinstanton, respectively. For more than 10 cooling steps both topological charge and chiral condensate begin to die out and eventually vanish. Combining the above finding of Fig. 1 showing that the correlation functions between $\bar{\psi}\psi(x)$ and $q^2(y)$ are not very sensitive to cooling together with the cooling history of the 3D images in Fig. 2, we conclude that instantons go hand in hand with clusters of $\bar{\psi}\psi(x) \neq 0$ also in the uncooled QCD vacuum [6].

Figure 2 has demonstrated that the quark-antiquark density attains its maximum values at the same positions where the extreme values of the topological charge density are situated. This behavior is further substantiated in Fig. 3 where the $\bar{\psi}\psi(x)$-values are plotted against $q(x)$ for all points x in the same configuration at 10 cooling steps. At first sight a linear relationship between the absolute value of the topological charge density and the virtual quark density is suggested.

In summary, our calculations of correlation functions between topological charge and the quark condensate yield an extension of about two lattice spacings. The correlations suggest that the local chiral condensate takes a non-vanishing value predominantly in the regions of instantons and monopole loops. It was well known before that the chiral condensate is related to the topological charge and topological susceptibility. The visualization exhibited that the distribution of the "chiral condensate" concentrates around areas with enhanced topological activity (instantons, monopoles). We demonstrated that exactly at those places in Euclidean space-time, where tunneling between the vacua occurs, amplified production of quark condensate takes place. It must be emphasized that this represents the situation on a finite lattice with finite quark mass without the extrapolation to the thermodynamic and chiral limit. We found for full $SU(3)$ QCD with dynamical quarks that the clusters of non-vanishing quark condensate have a size of about 0.4 fm, which corresponds to the instanton sizes observed in the same configurations. Visualization of quark and gluon fields might be especially useful to decide if the instanton-liquid model is realized in nature and if instanton-antiinstanton pairs appear in the deconfined phase. It might also help to clarify the question of the existence of a disoriented chiral condensate with consequences for heavy-ion experiments.

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